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TEMPERATURE CONTROL OF MULTI-INPUT MULTI-OUTPUT CROSS-COUPLED INDUSTRIAL OVENS

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ABSTRACT. *This paper considers the application of frequency domain techniques to the design of multivariable output feedback controllers for nonlinear, multiple-heating-zones industrial ovens. Specifically, the application of multivariable frequency domain techniques to a nonlinear oven with six heating zones is presented. Due to the oven construction, the heat flow rate among the zones was high and so it was the cross-coupling between inputs and outputs. The system decoupling is reached by using a pre-compensator tuned at 0 rd/min. Since the pre-compensated plant became clearly diagonal dominant at low frequencies, the system specification are reached using a MIMO diagonal PID controller. Multivariable frequency domain techniques are used to analyze the system, to obtain the system decoupling and to validate a single-input single-output (SISO) linear control design approach. Simulation results are presented.*

Key Words. *Multivariable Frequency Domain Control. Temperature Control.*

1. INTRODUCTION

Incorrect control tuning is one of the main causes for poor control performance and unnecessary economic cost in industry Bialkowski (1993). The difficulties in the control tuning procedures are direct consequence of inaccurate plant models, plant nonlinear characteristics and time delays, among others.

In the case of SISO systems, control parameters are frequently set to factory values or are manually tuned in a trial and error procedure. The reason for this is the lack of systematic procedures for tuning industrial controllers. In the MIMO case, the control tuning currently requires sophisticated techniques to reduce the usually existing cross interaction between inputs and outputs. This interaction obscures the effects of a specific loop controller on the system behavior. Thus, overcome the cross-coupling between inputs and outputs has become one of the primary objectives in multivariable control.

Several interesting multivariable control design techniques have been proposed by the scientific community, some of them are given in Desoer et al (1980), Doyle et al (1981), Edmunds et al (1979), Kouvaritakis et al (1979), MacFarlane et al (1977) and Mees, (1989). Traditionally, however, engineers in industry are used to the widely accepted standard tuning procedures for SISO systems. For this reason, tuning schemes for MIMO systems will succeed and be quickly accepted by the industry community as long as they are simple and similar to the ones for the SISO case. Using the same tuning algorithms, different people must arrive to the same values for the control parameters, this clearly requires a systematic tuning procedure.

In the case of temperature control, the control design problem is still more challenging due to the time delays and nonlinear parameters involved in the process. Multiple-input multiple- output industrial furnaces and ovens are among the most common industrial processes which still require improved and simple techniques for tuning output feedback controllers.

Large industrial ovens usually include several heating zones with individual sensors and actuators to control the output temperature profile. However, due to the heat flow among these zones there is a cross iteration among sensors and actuators. As in other cases of multiple-input multiple-output systems this iteration may become relevant and cause serious difficulties to the control system.

2. A BRIEF REVIEW

This section presents a brief review of the basic concepts on multivariable control systems, the following is based on the books from Maciejowski (1989) and Skogestad et al (1989):

In general, MIMO systems are represented by a matrix transfer function of the form:

$$y(s) = T(s)\ P(s)\ r(s) + S(s)\ d(s) - T(s)\ m(s) \tag{1}$$

where *r(s)* is the reference input, *d(s)* represents the disturbances and *m(s)* is the measurement noise.

In this case, $S(s)$ is known as the sensitivity matrix and is given by:

$$S(s) = [I + G(s)K(s)]^{-1} \tag{2}$$

T(s) is the system closed loop transfer function matrix (or complementary sensitivity function) and is given by:

$$T(s) = [I + G(s)K(s)]^{-1}\, G(s)\, K(s) = S(s)G(s)K(s) \tag{3}$$

also, the input sensitivity functions are defined as

$$\begin{aligned} S_i(s) &= [I + K(s)G(s)]^{-1} \\ T_i(s) &= [I + K(s)G(s)]^{-1}\, K(s)\, G(s) = K(s)G(s)S_i(s) \end{aligned} \tag{4}$$

Considering a multiplicative model for the plant uncertainty one has:

$$G(s) = G_0(s)[I + W_i(s)] \tag{5}$$

Then, from the literature , the following criteria to evaluate the system performance and stability can be established (Maciejowski, 1989), (Skogestad, 1989):

- The criterion for nominal performance is defined by

$$\left\| S(s)\ W_p(s) \right\|_\infty < 1 \tag{6}$$

where $W_p(s)$ is a performance weighting matrix.

In the case that $W_p(s)$ has the form

$$W_p(s) = w_p(s)\ [I] \tag{7}$$

the criterion for nominal performance becomes

$$\bar{\sigma}[S(s)] < \frac{1}{w_p(s)} \tag{8}$$

where $\bar{\sigma}[.]$ is the greatest singular value of [.]

- The criterion for robust performance (non structured uncertainty) is given by

$$\gamma\ \bar{\sigma}\left(w_p(s)\ S_i(s)\right) + \bar{\sigma}\left(w_i(s)\ T_i(s)\right) \leq 1 \tag{9}$$

where γ = min (plant condition number, controller condition number).

- The criterion for robust stability (non structured uncertainty) is defined by

$$\left\| T(s)\ W_i(s) \right\|_\infty < 1 \tag{10}$$

where $W_i(s)$ is an uncertainty weighting matrix.

In the case that $W_i(s)$ has the form

$$W_i(s) = w_i(s)\ [I] \tag{11}$$

the criterion for robust stability becomes

$$\bar{\sigma}[T(s)] < \frac{1}{w_i(s)} \tag{12}$$

- The robust performance condition for structured uncertainty, (Doyle et al, 1981), is given by

$$\mu\left(Q(s)\right) < 1 \qquad \forall \omega \tag{13}$$

where the matrix $Q(s)$ is defined as

$$Q(s) = \begin{bmatrix} w_p(s)\ S_0(s) & w_p(s)\ S_0(s)\ G_0(s) \\ -w_i(s)K(s)S_0(s) & -w_i(s)K(s)S_0(s)G_0(s) \end{bmatrix} \tag{14}$$

with

$$S_0(s) = \left(I + G_0(s)\ K(s)\right)^{-1} \tag{15}$$

- The robust stability condition for structured uncertainty Doyle et al (1981), is given by

$$\mu\left(Q_{22}(s)\right) < 1 \qquad \forall \omega \tag{16}$$

where

$$Q_{22}(s) = -w_i(s)K(s)S_0(s)G_0(s) \tag{17}$$

3. THE PLANT

The case studied is a tubular-shaped electrical oven with six inputs and six outputs, and six meters long. The control objective is to regulate the temperature profile (Galvez et al, 1995). A major challenge in this case is that the construction specifications work against the control performance: To improve the smoothness of the temperature profile, the iteration (heat flow) among the heating zones must be as large as possible which causes the undesirable cross-coupling among sensors and actuators.

The saturation characteristics of the power source and the two different time constants for heating and cooling make the system a nonlinear one. However, it was shown in Galvez et al (1995) that for some temperature profiles the system behaves linearly allowing linear analysis and design. Dynamic tests were performed to obtain and validate a linear model. Based on experimental results, a simplified tri diagonal matrix transfer function model was built such that

$$Y(s) \quad = \quad G(s)\ U(s) \tag{18}$$

where

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & 0 & 0 & 0 & 0 \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & 0 & 0 & 0 \\ 0 & G_{32}(s) & G_{33}(s) & G_{34}(s) & 0 & 0 \\ 0 & 0 & G_{43}(s) & G_{44}(s) & G_{45}(s) & 0 \\ 0 & 0 & 0 & G_{54}(s) & G_{55}(s) & G_{56}(s) \\ 0 & 0 & 0 & 0 & G_{65}(s) & G_{66}(s) \end{bmatrix} \tag{19}$$

with

$$G_{ii}(s) = \frac{10.8\ e^{-\ t}}{68.7\ s+1} \quad \& \quad G_{ij}(s) = \frac{5.25\ e^{-25\ t}}{108.33\ s+1} \tag{20}$$

and

$$Y(s) = [y_1(s) \quad y_2(s) \quad y_3(s) \quad y_4(s) \quad y_5(s) \quad y_6(s)]^T \tag{21}$$

$$U(s) = [u_1(s) \quad u_2(s) \quad u_3(s) \quad u_4(s) \quad u_5(s) \quad u_6(s)]^T \tag{22}$$

Figure 1 shows the frequency response of the $G_{ii}(s)$ transfer function (main diagonal). Notice that the cross-over frequency (0 dB) is around 0.15 rd/min. Figure 2 shows partial Bode diagrams of the open loop system.

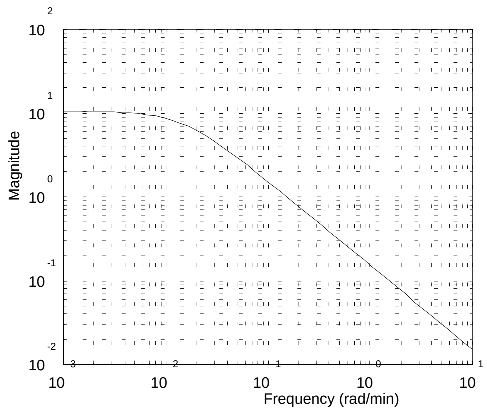

Figure 1. Frequency response of $G_{ii}(s)$.

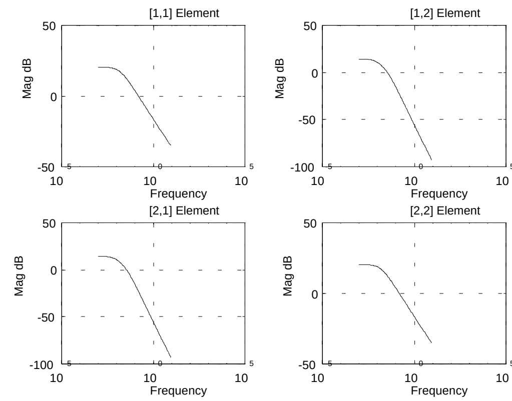

Figure 2. Bode diagrams of the open loop system.

Figure 3 shows (from bottom to top) the open loop step response for $G_{22}(s)$ (Zone 2) with no cross-coupling, one-side cross-coupling and two-side cross-coupling effects.

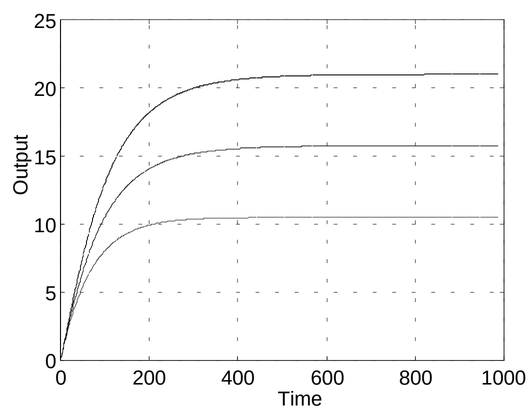

Figure 3. The open loop step response (Zone 2).

4. THE CONTROLLER

Output feedback control has been extensively applied to industrial processes basically because its design does not requires an exact model for the plant dynamics. To determine the controller structure and its parameters, some design techniques in the time domain set the controller design problem as an optimization problem. In these approaches the practical aspects of the control problem are neglected in exchange for convenience in the theoretical formulation and numerical solution of the optimization problem.

On the other hand, multivariable frequency domain methods are basically a sophisticated generalization of the classical SISO frequency domain techniques and so they are usually implemented following a trial and error procedure.

It should be noted, however, that behind the success of most design techniques, for output feedback control, resides a natural property of dynamic systems: the pole dominance concept. Even the most simple PID control design technique intrinsically uses this concept to determine the controller parameters. In practice, the concept also allows the control manual tuning with a relatively high rate of success. In this work, the pole dominance concept is used to decouple the MIMO system at 0 rd/min and then a MIMO diagonal PID controller is designed. To validate the controller design, multivariable techniques are used following a more application-oriented approach. In the case studied, the steady state temperature profile was the main control objective. So, the system was required to fulfill two basic specifications, a fast transient response and a null steady state error. Figure 4 shows the system block diagram.

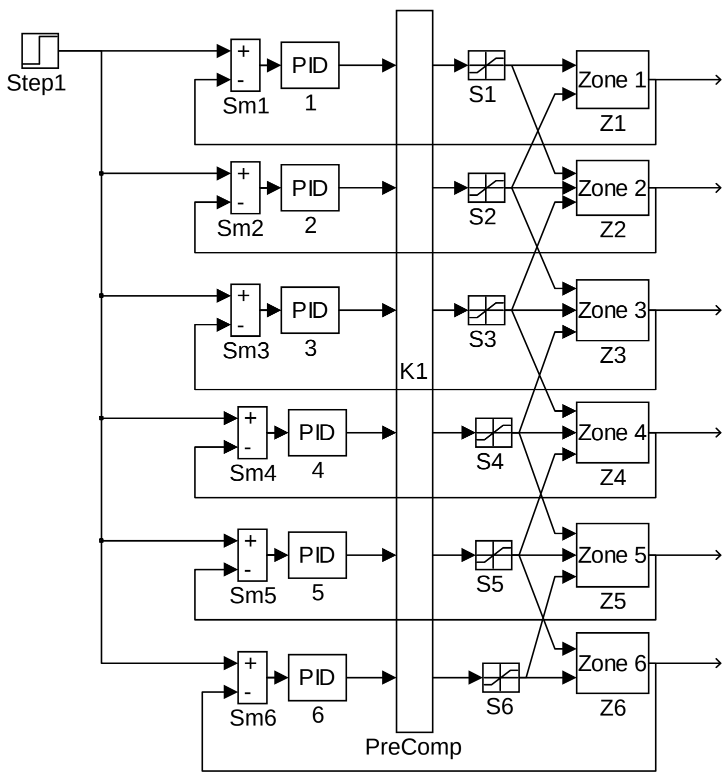

Figure 4. The system block diagram.

To deal with low frequency disturbances and to obtain good tracking of step inputs the plant is required to be a type-1 system. Thus a performance weighting function was defined as:

$$W_p(s) = \left(\frac{50\ s+1}{100\ s}\right)[I] = 0.5\left(\frac{s+0.02}{s}\right)[I] \tag{23}$$

Considering the overall characteristics of the system, the maximum system gain drift is expected to be less than 15 %. Then, the uncertainty weighting function was defined as:

$$W_i(s) \;=\; 0.15\,\left(\frac{s+1}{0.1\,s+1}\right)\,[I] \;=\; 1.5\left(\frac{s+1}{s+10}\right)\,[I] \tag{24}$$

To ensure a good steady state performance, the closed-loop system requires low frequency decoupling at 0 rd/min Galvez et al (1995), Kouvaritakis et al (1979), MacFarlane et al (1977) and Mees (1989). Then, the decoupling pre-compensator can be computed as

$$K_1 = G(0)^{-1} \cong \begin{bmatrix} +0.1467 & -0.1113 & +0.0822 & -0.0579 & +0.0368 & -0.0179 \\ -0.1113 & +0.2289 & -0.1692 & +0.1191 & -0.0758 & +0.0368 \\ +0.0822 & -0.1692 & +0.2658 & -0.1871 & +0.1191 & -0.0579 \\ -0.0579 & +0.1191 & -0.1871 & +0.2658 & -0.1692 & +0.0822 \\ +0.0368 & -0.0758 & +0.1191 & -0.1692 & +0.2289 & -0.1113 \\ -0.0179 & +0.0368 & -0.0579 & +0.0822 & -0.1113 & +0.1467 \end{bmatrix} \tag{25}$$

Figure 5 shows the test functions for nominal performance ($1/w_p$) and robust stability ($1/w_i$). Figure 6 presents partial Bode diagrams of the open loop pre-compensated system. It can be seen that the cross-coupling gains became neglected at low frequencies and also that the system pass band was substantially reduced by the pre-compensation.

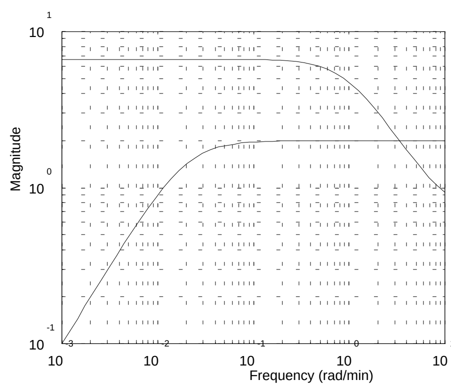

Figure 5. The nominal performance ($1/w_p$) and robust stability ($1/w_i$) tests.

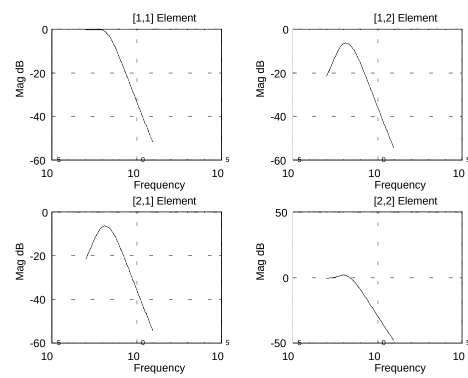

Figure 6. Bode diagrams of the pre-compensated open loop system.

Since the pre-compensated system has became diagonal dominant at low frequencies, then a diagonal PID controller can be designed to set the steady state error to zero. Thus, a multivariable controller can be built as $K(s) = K_1\ K_2(s)$ and the open loop transfer function matrix becomes $G(s)\ K(s) = G(s)\ K_1\ K_2(s)$

Figure 7 shows the principal gains of the open-loop system (controller + plant). Figure 8 shows the nominal performance criterion corresponding to Equation 8. Figure 9 shows the robust performance condition for non structured uncertainty given by Equation 9. Figure 10 shows the robust stability criterion for non structured uncertainty given by Equation 12. Figure 11 shows the μ(Q) value for robust performance (Equation 13). Figure 12 shows the $\mu(Q_{22})$ value for robust stability (Equation 16).

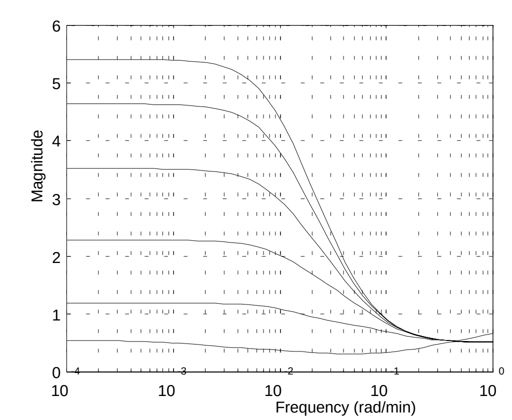

Figure 7. The principal gains of the open-loop system (controller + plant).

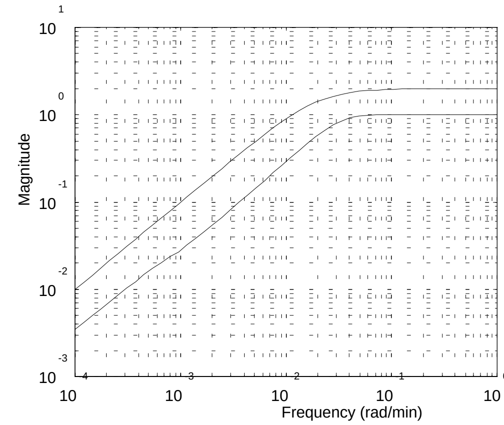

Figure 8. The nominal performance condition given by Equation 8 (NSU).

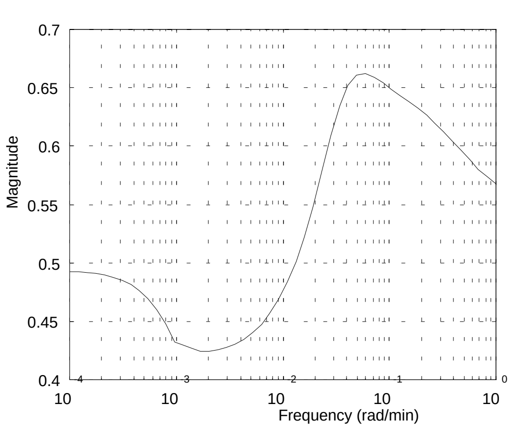

Figure 9. The robust performance condition given by Equation 9 (NSU)..

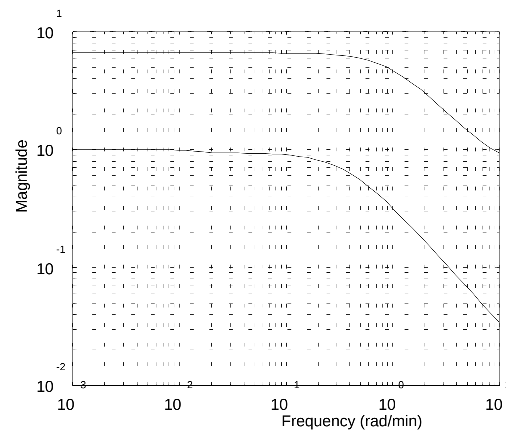

Figure 10. The robust stability condition given by Equation 12, (NSU).

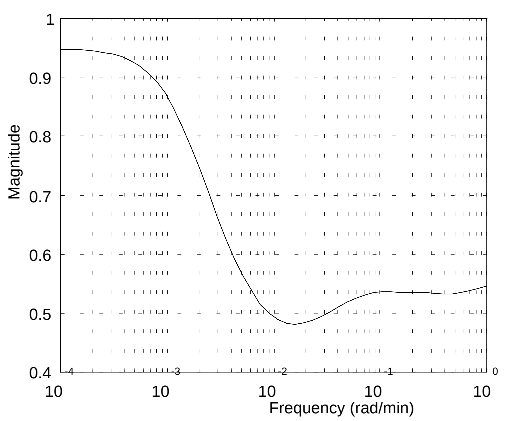

Figure 11. The $\mu(Q)$ condition for robust performance, Equation 13, (SU).

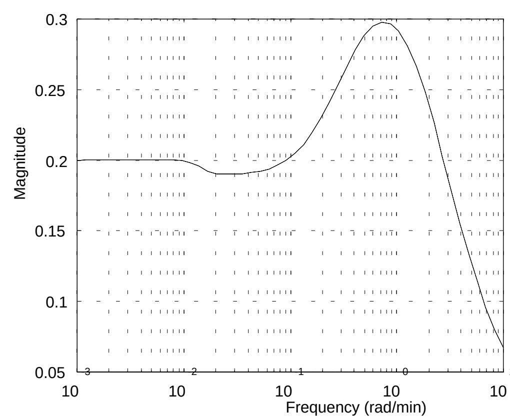

Figure 12. The $\mu(Q_{22})$ condition for robust stability, Equation 16, (SU).

5. SIMULATION RESULTS

Simulation experiments were performed to validate the multivariable control design. The tri-diagonal model used in the numerical simulations includes the saturation characteristics of the power source. Due to the unidirectional power source, negative power values can not be applied in practice. Usually, this causes a high level of performance deterioration on the multivariable pre-compensator and controller which are designed based on a linear model of the plant. However, it was shown in Galvez et al (1995) that, in this case, the system behaves linearly under the control design specifications.

Figure 13 presents simulation results for the closed loop pre-compensated system (Zones 1, 2 and 3). The multivariable diagonal PI controller ($K_{2a}(s)$) was designed to satisfy the stability and robustness specifications given in the previous section (K_p = 3.0 and K_i = 0.0125). This choice for the control parameters delivered a time response with a settling time of t_s = 200 minutes and damping ratio of ξ = 0.8.

$$K_{2a} = K_p \left(1 + \frac{K_i}{s}\right)[I]; \qquad K_p = 3.0,\ K_i = 0.0125 \tag{26}$$

Figure 14 presents simulation results for the closed loop pre-compensated system (Zones 1, 2 and 3). In this case a multivariable diagonal PID controller ($K_{2b}(s)$) was manually tuned (K_p = 6.5, K_i = 0.015 and K_d = 1.25) to obtain a rise time of t_r = 40 minutes with a damping ratio of ξ = 0.5.

$$K_{2b} = K_p \left(1 + \frac{K_i}{s} + K_d\ s\right)[I]; \qquad K_p = 6.5,\ K_i = 0.015,\ K_d = 1.25 \tag{27}$$

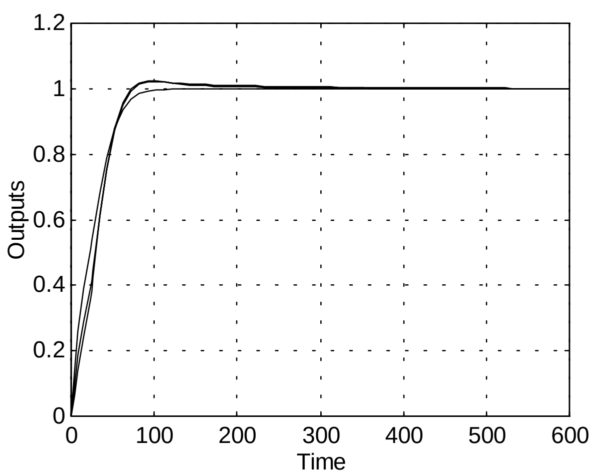

Figure 13. The pre-compensated closed loop system step response with PI controller.

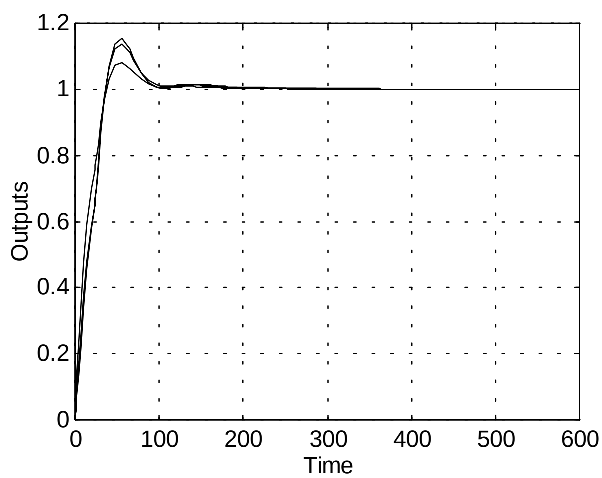

Figure 14. The pre-compensated closed loop system step response with PID controller.

6. FINAL COMMENTS

An interesting application of multivariable frequency domain techniques to a nonlinear electrical oven with six heating zones has been presented in this work. Due to the oven construction, the heat flow rate among the zones was high and so it was the cross-coupling between inputs and outputs.

It was verified that for some temperature profiles, the nonlinear characteristics of the power source can be avoided allowing a linear analysis and design. The system decoupling was easily reached by using a pre-compensator tuned at 0 rd/min. Since the pre-compensated plant became clearly diagonal dominant at low frequencies, the system specification were reached using a diagonal PID controller.

Multivariable frequency domain techniques were used to validate the controller. However, both conditions for robust performance under unstructured and structured uncertainty (Equations 9 and 13) gave conservative results as it was verified in simulation and in practice.

Finally, a general procedure for control design can be suggested as follows:

- Find the optimal cross-coupling gains and modify, if possible, the plant gains to those values. Compute a decoupling gain matrix tuned at 0 rd/min.
- Design a multivariable feedback controller, such as a lag type diagonal controller, tuned to met the performance specifications.
- Whether high frequency compensation is required, compute a decoupling controller at the desired frequency and introduce a lead type controller to match the system specifications.
- Validate the controller design by some multivariable technique.

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